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A note on 'Lifting three-dimensional wings in transonic flow' by M. S. Cramer

By M. S. CRAMER

Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

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The purpose of this note is to clarify the discrepancy between the results of Cramer (1979), Barnwell (1975) and Cheng & Hafez (1975). These authors have all derived the boundary-value problem governing the flow far from a three-dimensional lifting wing in transonic flow. The results of both Cramer and Barnwell will provide the *lowest-order* solution in the far field. The results of Cheng & Hafez are not only accurate to lowest order but to *first order* as well. That is, the theory of Cheng & Hafez is an order-of-magnitude more accurate than those of Barnwell and Cramer. This is the reason for the discrepancy between the boundary condition derived by Cheng & Hafez and that of Cramer and Barnwell. Because Cheng & Hafez correctly derive the first-order results, the lowest-order theories of Cramer or Barnwell cannot correct those of Cheng & Hafez. It should be recognized that the main contribution of Cramer's work was to clarify the structure of the problem and to simplify the analysis rather than correct the basic results.

The straightforward perturbation procedure used by Cramer can, of course, be used to obtain the first-order problem for the far-field flow. This is seen to be in complete agreement with the results of Cheng & Hafez once it is recognized that the latter's Φ' is really the first *two* terms of the asymptotic expansion of the velocity potential valid in the far field. In the notation of Cheng & Hafez, this could be written

$$\Phi' = \Phi_0 + \epsilon' \Phi_1.$$

In the course of the analysis, one not only needs to verify that the appropriate boundary condition for Φ' is just equation (4.12), but that the quantity Φ' still satisfies the classical transonic small-disturbance equation (4.10), at least to the appropriate order. It should also be remembered that Barnwell's procedure could also be used to derive the correct first-order problem for the outer flow. In this case, one needs to take into account the nesting of the logarithmic terms.

In conclusion, the three previous investigations provide us with the correct lowest-order boundary-value problem for the flow in the far field. An inspection of higher-order terms shows that the lowest-order theory is a completely consistent one; that is, solutions obtained from it are expected to agree well with the appropriate experimental and numerical studies. In addition to this, Cheng & Hafez have derived the first correction to these lowest-order theories.

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